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### Brief Outline of Research Findings

Work continued on the thermodynamics of two-phase continua. The general two-phase Stefan problem with supercooling, superheating, and capillarity, was studied [T19] in collaboration with M. Soner (Carnegie Mellon). Simple solutions – illustrating the chief differences between this problem and the classical Stefan problem – were obtained for the spherically symmetric problem, under the assumption of fast diffusion, with the liquid supercooled at infinity. It is shown that: (i) for  $\Omega = \mathbb{R}^3$ , a ball of the solid phase of sufficiently small size disappears in finite time, but a sufficiently large ball grows without bound; (ii) for  $\Omega = \mathbb{R}^3$  and the solid phase initially situated in a spherical shell of thickness  $\epsilon$ , the thickness of the solid shell initially increases, but the inner radius of this region decreases to zero in finite time  $T$ ; the solid ball remaining at time  $T$  disappears at a later time or grows without bound according as  $\epsilon$  is less than or greater than a critical value; in the limit  $\epsilon \rightarrow 0$  the region occupied by the solid disappears infinitely fast; the problem has no solution for  $\epsilon = 0$ ; (iii) when  $\Omega$  is the region exterior to a sphere of radius  $R$ , with the boundary  $r = R$  insulated and with the solid phase initially in a spherical shell of zero thickness at  $r = R$ , the solid phase grows without bound provided  $R$  is sufficiently large. While (ii) and (iii) are of little practical interest, they demonstrate the possibility of growth from a seed of zero volume.

Papers and reports which appeared under this contract

- [T1]\* Gurtin, M.E., Multiphase thermomechanics with interfacial structure. 1. Heat conduction and the capillary balance law. *Archive for Rational Mechanics and Analysis*, 104, 195–221 (1988).
- [T2]\* Gurtin, M.E., On a nonequilibrium thermodynamics of capillarity and phase, *Quarterly of Applied Mathematics*, 97, 129–145 (1989).
- [T3]\* Gurtin, M.E., On diffusion in two-phase systems: the sharp interface versus the smooth transition layer, *Proceedings CNRS-NSF Symposium*, Nice (1988), Springer Lecture Notes in Physics 344, (1989).
- [T4]\* Browning, R.V., M.E. Gurtin, and W.O. Williams, A model for viscoplastic materials with temperature dependence, *International Journal of Solids and Structures*, 25, 441–457 (1989).
- [T5]\* Almgren, F. and M.E. Gurtin, A mathematical contribution to Gibbs's analysis of fluid phases in equilibrium, *Partial Differential Equations and the Calculus of Variations, Volume I, Essays in Honor of Ennio De Giorgi*, (eds. F. Colombini, A. Marino, L. Modica, S. Spagnolo) Birkhauser, Boston, 1989.
- [T6]\* Milic, N., On non-equilibrium phase transitions in mixtures with interfacial structure, Ph.D. Thesis, Department of Mathematics, Carnegie Mellon University (December 1988).
- [T7]\* Gurtin, M.E., On the isothermal motion of a phase interface, *Proceedings of the Conference (1988) "Problems Involving Change of Type"*, *Lecture Notes in Physics*, 359 (ed. K. Kirchgassner) Springer-Verlag, Berlin, 1990.
- [T8]\* Gurtin, M.E., A. Struthers, and W.O. Williams, A transport theorem for moving interfaces, *Quarterly of Applied Mathematics*, 47, 773–777 (1989).
- [T9]\* Angenent, S. and M.E. Gurtin, Multiphase thermomechanics with interfacial structure. 2. Evolution of an isothermal interface, *Archive for Rational Mechanics and Analysis*, 108, 323–391 (1989).
- [T10]\* Gurtin, M.E., A mechanical theory for crystallization of a rigid solid in a liquid melt: melting-freezing waves, *Archive for Rational Mechanics and Analysis*, 110, 287–312 (1990).  
—
- [T11]\* Gurtin, M.E. and P. Podio-Guidugli, A hyperbolic theory for the evolution of plane curves, *SIAM Journal on Mathematical Analysis*, 22, 575–586 (1991).
- [T12]\* Gurtin, M.E. and W.J. Hrusa, On the thermodynamics of viscoelastic materials of single-integral type, *Quarterly of Applied Mathematics*. Forthcoming.

- [T13]\* Gurtin, M.E., On the continuum mechanics of the motion of a phase interface. *Proceedings Seventh Army Conference on Applied Mathematics and Computing*, West Point (1989).
- [T14]\* Gurtin, M.E. and A. Struthers, Multiphase thermodynamics with interfacial structure. 3. Evolving phase boundaries in the presence of bulk deformation, *Archive for Rational Mechanics and Analysis*, 112, 97–160 (1990).
- [T15]\* M.E. Gurtin and W.J. Hrusa, Global existence in one-dimensional nonlinear viscoelasticity with heat conduction, *Journal of Integral Equations*, 2, 431–460 (1990).
- [T16]\* F. Davi and M.E. Gurtin, On the motion of a phase interface by surface diffusion, *Zeitschrift für angewandte Mathematik und Physik*, 41, 782–811 (1990).
- [T17]\* M.E. Gurtin, On thermomechanical laws for the motion of a phase interface, *Zeitschrift für angewandte Mathematik und Physik*. Forthcoming.
- [T18]\* Gurtin, M.E., Evolving phase boundaries in the presence of deformation and surface stress, *Proceedings IMA Workshop on Evolving Phase Boundaries*, U. Minnesota (1990).
- [T19] Gurtin, M.E. and H.M. Soner, Some remarks on the Stefan problem with surface structure, *Quarterly of Applied Mathematics*. Forthcoming.

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\* Discussed in previous progress reports.